ALLOWANCE FOR THE CHARACTERISTICS OF THE BED AND SATURATING FLUID IN PERCOLATION PROCESSES IN DEFORMABLE RESERVOIRS

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Many oil deposits confined to fissured reservoirs have now been opened up and are being exploited. They are characterized, on the one hand, by their deformability and, on the other, by the anisotropicity of their percolation properties. This accounts for the importance of the study of percolation through anisotropic fissured rocks with allowance for their deformability. The relationship between two important characteristics, permeability tensor and fictitious stress tensor, and in the general case of a rock anisotropic with respect to its elastic properties, the elastic modulus tensor, is considered. It is shown that these relations are nonlinear in form and can be replaced with the linear relations ordinarily employed only in the case of small pressure changes. Formulas for the permeability in an isotropic deformable or anisotropic nondeformable bed follow from the equations obtained as special cases.

From the work of Ferrandon [1], Scheidegger [2], and E. S. Romm [3] it is known that the permeability tensor of a nondeformable fissured medium has the form

$$k_{rs} = \frac{1}{12}\sum_{i=1}^{N} m_{*i} b_i^2 (\delta_{rs} - \alpha_{ri}\alpha_{si}) \qquad (r, s = 1, 2, 3). \tag{1}$$

Here, $m_{*i} = a\Gamma_i b_i$ is the crack porosity of the i-th crack system; $\Gamma_i b_i$ are the density and openness of the i-th system of cracks; δ_{rs} are the components of the unit tensor; and α_{ri}, α_{si} are direction cosines.

It is quite understandable that, in the case of a change of formational pressure in the liquid saturating a fissured bed, the permeability k_{rs} will vary as a result of both the change in m_{*i} and the change in b_i.

Accordingly, from Eq. (1) there follows:

$$\frac{dk_{rs}}{k_{rs}} = \frac{dm_*^\circ}{m_*^\circ} + 2\frac{\Sigma\lambda_i db_i}{\Sigma\lambda_i b_i}, \quad \lambda_i = \xi_i\varphi_i(\delta_{rs} - \alpha_{ri}\alpha_{si}),$$

$$\frac{dm_*^\circ}{m_*^\circ} = \frac{d[\Sigma m_{*i}\xi_i^2(\delta_{rs} - \alpha_{ri}\alpha_{si})]}{\Sigma m_{*i}\xi_i^2(\delta_{rs} - \alpha_{ri}\alpha_{si})} \qquad \left(\xi_i = \frac{b_i}{b_0}\right). \tag{2}$$

Here, $\xi_i = b_i/b_0$ is the correction for the nonuniform openness of different cracks; φ_i is the correction for the nonuniform porosity of cracks of different systems; and Σ denotes summation with respect to i; in what follows this condition is always assumed to be satisfied.

We transform (2), introducing the ratio

$$\frac{dV_*}{V_*} = \frac{d(\Sigma n_i b_i l_i a_i)}{\Sigma n_i b_i l_i a_i}. \tag{3}$$

Here, n_i, l_i, and a_i are, respectively, the number, length, and width of cracks with openness b_i. Following Fatt [4], we express the width and length of a crack in the form of a function of b_i to certain powers α and β, i.e.,

$$l_i = c_1 b_i^\alpha, \qquad a_i = c_2 b_i^\beta.$$

Then Eq. (3) takes the form:

$$\frac{dV_*}{V_*} = \frac{\Sigma n_i C(\alpha+\beta+1) b_i^{\alpha+\beta} db_i}{\Sigma n_i C b_i^{\alpha+\beta+1}} =$$

$$= (\alpha+\beta+1)\frac{\Sigma\eta_i\xi_i^{\alpha+\beta}db_i}{\Sigma\eta_i\xi_i^{\alpha+\beta}b_i}. \tag{4}$$

Here, α, β are structural coefficients; η_i is a correction for the nonuniformity of the number of cracks of the i-th system, for the nonuniformity of c_i, etc.

We assume that

$$\eta_i\xi_i^{\alpha+\beta} = \nu\lambda_i^\gamma \qquad (\nu, \gamma = \text{const}). \tag{5}$$

Then from (4) we have

$$\frac{dV_*}{V_*} = (\alpha+\beta+1)\frac{\Sigma\lambda_i^\gamma db_i}{\Sigma\lambda_i^\gamma b_i} = (\alpha+\beta+1)\,\theta^{\gamma-1}\frac{\Sigma\lambda_i db_i}{\Sigma\lambda_i b_i},$$

$$\theta^{\gamma-1} = \frac{\Sigma\lambda_i^\gamma db_i}{\Sigma\lambda_i db_i} \Bigg/ \frac{\Sigma\lambda_i^\gamma b_i}{\Sigma\lambda_i b_i}. \tag{6}$$

For the crack porosity we have

$$m_* = V_*/V, \tag{7}$$

where V_* is the volume of the cracks, and V the volume of the bed; consequently,

$$\frac{dm_*}{m_*^\circ} = \frac{dV_*}{V_*} - \frac{dV}{V} \quad \text{or} \quad \frac{dV_*}{V_*} = \frac{dm_*}{m_*(1-m_*)}. \tag{8}$$

Combining (8) and (6), we obtain

$$\frac{\Sigma\lambda_i db_i}{\Sigma\lambda_i b_i} = \frac{1}{(1+\alpha+\beta)\,\theta^{\gamma-1}}\frac{dm_*}{m_*(1-m_*)}. \tag{9}$$

Substituting (9) into (2), we obtain

$$\frac{dk_{rs}}{k_{rs}} = \frac{dm_*^\circ}{m_*^\circ} + \frac{2}{(1+\alpha+\beta)\,\theta^{\gamma-1}}\frac{dm_*}{m_*(1-m_*)}. \tag{10}$$

The first terms on the right-hand side can be transformed:

$$\frac{dm_*^\circ}{m_*^\circ} = \frac{d[\Sigma m_*\lambda/\varphi_i]}{\Sigma m_{*i}\lambda_i/\varphi_i} = \frac{dm_*\Sigma\lambda_i}{m_*\Sigma\lambda_i} = \frac{dm_*}{m_*}.$$

Equation (10) takes the form:

$$\frac{dk_{rs}}{k_{rs}} = \frac{dm_*}{m_*} + \frac{2}{(1+\alpha+\beta)\,\theta^{\gamma-1}}\frac{dm_*}{m_*(1-m_*)}. \tag{11}$$

Integrating expression (11), for $\theta = 1$ we obtain

$$\frac{k_{rs}}{k_{rs0}} = \left(\frac{m_*}{m_{*0}}\right)\left(\frac{m_* - m_* m_{*0}}{m_{*0} - m_* m_{*0}}\right)^\vartheta \quad \left(\vartheta = \frac{2}{1+\alpha+\beta}\right). \tag{12}$$

Usually in fissured reservoirs $m_{*0} \gg m_{*0} m_*$; therefore, neglecting the product $m_* m_{*0}$, from (12) we find

$$k_{rs} = k_{rs_0}\left(\frac{m_*}{m_{*0}}\right)^\vartheta \qquad \left(\vartheta = \frac{3+\alpha+\beta}{1+\alpha+\beta}\right). \tag{13}$$

Here, k_{rs_0} and m_{*0} are the permeability and porosity at $P = P_0$. It is easy to see that if $\alpha = 0$, $\beta = 0$, i.e., $l_i = c_1$, $a_i = c_2$ and are constant, which corresponds to cracks of constant width and length running through the entire bed, then

$$k_{rs} = k_{rs_0}\left(\frac{m_*}{m_{*0}}\right)^3. \tag{14}$$

Assuming that in a medium isotropic with respect to the elastic properties [5, 6]

$$\frac{m_*}{m_{*0}} = 1 + c(P - P_0) + d_{rs}\Delta\sigma_{rs}', \tag{15}$$

where c and d_{rs} are the force constants, after substituting into (13) we obtain

$$k_{rs_0} = k_{rs}[1 + c(P - P_0) + d_{rs}\Delta\sigma^f_{rs}]^{\vartheta}. \quad (16)$$

For a medium anisotropic with respect to elastic properties we correspondingly have

$$k_{rs} = k_{rm_0}[\delta_{ms} + c_{ms}(P - P_0) + d_{klms} \Delta\sigma^f_{kl}]^{\vartheta}. \quad (17)$$

Here, c_{ms} and d_{klms} are the force constants with 9 and 45 components, respectively (tensor symmetrical with respect to two indices).

Equation (17) can be written in the following general form:

$$K = K_0[I + C(P - P_0) + D\Delta\Psi^f]^{\vartheta}. \quad (18)$$

If it is assumed that the tensor T_{kl} of the general state of stress due to the external load is a constant, Eq. (17) takes the form:

$$k_{rs} = k_{rm_0}[\delta_{ms} + c_{ms}(P - P_0) - d_{klms}\delta_{kl}(P - P_0)]^{\vartheta}, \quad (19)$$

or

$$k_{rs} = k_{rm_0}[\delta_{ms} - \lambda_{ms}(P_0 - P)]^{\vartheta},$$

$$\lambda_{ms} = d_{klms}\delta_{kl} - c_{ms}. \quad (20)$$

As may be seen from (20), the relation obtained in this case is consistent with Tiller's formula [7], about which Scheidegger wrote [8], if one takes into account that $\sigma_{kl} = T_{kl} - P\delta_{kl}$.

We also note that without disturbing the generality of (20) it is possible to obtain a relation for the permeability of a deformable fissured reservoir isotropic with respect to the elastic and percolation properties in the form $k = k_0[1 - \lambda(P_0 - P)]^3$, which coincides with the formulas in [9].

The advantages of a relation of type (20) are obvious, since it relates such important characteristics of the reservoir as the anisotropicity of the percolation and elastic properties.

In order to obtain a similar dependence for a porous reservoir, we start from Marshall's model [10], for which, taking into account the orientation of the cylindrical pores (channels), we introduce a correction coefficient. We then obtain

$$k_{rs} = \frac{m^2}{8N^2}\,\Sigma\,(2i - 1)\,r_i^2\,(\delta_{rs} - \alpha_{ri}\alpha_{si}). \quad (21)$$

We recall that here Σ represents summation with respect to the index i.

Proceeding as before, after intermediate transformations and representing the length of the pores in the model in the form $l_i = cr_{i\alpha}$, we obtain

$$\frac{k_{rs}}{k_{rs_0}} \approx \left(\frac{m}{m_0}\right)^{\varepsilon} \cong [1 + c(P - P_0) + d_{rs}\Delta\sigma_{rs}^f]^{\varepsilon}. \quad (22)$$

For a porous reservoir anisotropic with respect to the elastic properties

$$k_{rs} = k_{rm}[\delta_{ms} - \lambda_{ms}(P_0 - P)]^{\varepsilon} \qquad \left(\varepsilon = 2\frac{3 + \alpha}{2 + \alpha}\right). \quad (23)$$

We note that Marshall's relation was used by Dobrynin in [11] in establishing a relationship between the permeability and pressure for a porous reservoir isotropic with respect to percolation and elastic properties.

Generalizing the above, we note that, as a rule, a nonlinear relationship exists between the permeability tensor, the force constants and the fluid pressure.

The representation of these relations by means of a linear approximation is in many cases too rough an approximation, permissible in studying small permeability measurement intervals. In the case of a wider range of permeability variation it is necessary to use nonlinear relations.

On the basis of (20) we can write the law of percolation in a deformable anisotropic fissured reservoir in the form:

$$v = -\frac{K_0[I - \Lambda(P_0 - P)]^{\vartheta}}{\mu}\,\mathrm{grad}\,P. \quad (24)$$

Projecting onto the axes of the Cartesian coordinates, we obtain (and similarly for v_y and v_z)

$$v_x = -\left\{\frac{k_{xx_0}[1 - \lambda_{xx}(P_0 - P)]^{\vartheta}}{\mu}\frac{\partial P}{\partial x} + \frac{k_{xy_0}[1 - \lambda_{xx}(P_0 - P)]^{\vartheta}}{\mu}\frac{\partial P}{\partial y} + \frac{k_{xz_0}[1 - \lambda_{xx}(P_0 - P)]^{\vartheta}}{\mu}\frac{\partial P}{\partial z}\right\}$$

$$\lambda_{xx} = (d_{xxxx}\delta_{xx} - c_{xx}) + (d_{yyxx}\delta_{yy} - c_{xx}) + (d_{zzxx}\delta_{zz} - c_{xx}). \quad (25)$$

If as the coordinate axes we select the principal axes of the permeability tensor k_0, expression (25) takes the form:

$$v_x = -\frac{k'_{xx_0}[1 - \lambda_{xx}(P_0 - P)]^{\vartheta}}{\mu}\frac{\partial P}{\partial x} \quad (26)$$

and similarly for v_y and v_z. Here, k'_{xx} is the principal value of the tensor K_0.

In the principal axes of K_0 the flow continuity equation then assumes the form:

$$\nabla_i\left\{\frac{k'_{i_0}\rho}{\mu}[1 - \lambda_{ii}(P_0 - P)]^{\vartheta}\nabla_i P\right\} = \frac{\partial(m_*\rho)}{\partial t}, \quad (i = 1, 2, 3). \quad (27)$$

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